U. S. DEPARTMENT OF AGRICULTURE
Statistical Reporting Service

Variance Analysis of 1970 Wisconsin State Farm Census 1/

A variance analysis of data collected in the 1970 State Farm Census in Wisconsin was made primarily to develop resource material that would be useful for training in sampling. Hence, most of the interpretation is left for students and instructors in sampling or readers with a background in sampling theory and agriculture. To illustrate applications of sampling theory numerous exercises for students can be formulated with reference to the tables. Also, much time can be spent profitably on studying the patterns of variation portrayed by the tables and on examining the effectiveness of various sample designs in relation to the patterns of variation.

One of the reasons for selecting Wisconsin was its wide geographic variation in agriculture. Secondly, the data happened to be conveniently available on magnetic tape and the variances could be computed for a modest cost. Third, as a farm in the annual census in Wisconsin is identified by a number that remains unchanged from year-to-year there was an attractive potential for taking another year's data and studying variances with reference to year-to-year changes, which could add a very important dimension for study.

Items were selected for this study primarily with reference to variability characteristics from the viewpoint of sample design. A major criterion for selection of items was percent reporting which ranges from less than one percent for potatoes and snap beans to 100 percent for farmland. Another criterion was geographic distribution. Some items selected are more uniformly distributed over the State than others. Population (number of persons living on a farm) was included because the variation from farm to farm is low and it is an item reported by nearly all farms.

The numbers in columns (2) thru (6) of tables 1 and 2 are not official estimates. They are totals as enumerated in the Wisconsin State Farm Census and may differ from official estimates for several reasons including, under or over enumeration, definitions, and dates to which the data relate.

All variances in the accompanying tables are expressed as relative variances on a single unit basis, i.e., a variance can be interpreted as applying to a sample of size "one"--one farm in tables 1, 2, 3, and 4 and one township in table 6. Variance formulas are presented in appendix A so there will be no misunderstanding of what the variances are arithmetically. Appendix A gives explanations by columns of the tables.

1/ Prepared by Earl E. Houseman, December 1971.

Wed. Mat. 16

Crop Reporting Districts, CRD's, are subdivisions of the State which are used for various statistical purposes. They are relatively homogeneous groups of counties. See figure 1 at the end of the tables for an outline of the State showing CRD's and counties.

Townships are subdivisions of counties. A few townships had only one or two farms. Townships with less than four farms were combined with adjacent townships, giving a total of 1,462 townships (or township combinations) for purposes of this study. The average number of farms per township was 69.5

The system designed for processing the data involved two computer runs. The first run provided an output tape with the following results for each township, county, CRD, and the State:

For farmland: $N, \Sigma Y, \Sigma Y^2$, and S_y^2

where N is the number of farms,

Y is the number of acres of farmland, and

S_y^2 is the variance of farmland

For each item other than farmland: $N_n, \Sigma X, \Sigma X^2, \Sigma XY, S_X^2, S_{XY}$, and $S^2(\bar{Y}\frac{X}{Y})$

where N_n is the number of farms reporting the item (that is, N_n is the number with $X_i > 0$)

X represents any one of the selected items,

S_X^2 is the variance of X,

S_{XY} is the covariance of X and Y, and

$$S^2\left(\bar{Y}\frac{X}{Y}\right) = S_X^2 + \left(\frac{\Sigma X}{\Sigma Y}\right)^2 S_Y^2 - 2\left(\frac{\Sigma X}{\Sigma Y}\right) S_{XY}$$

which is the variance of $\bar{Y}\frac{X}{Y}$

Tables 1 and 2 were compiled from a print out of the CRD and State data on the output tape from the first run. The output tape from the first run was the input tape for the second run which gave results for the remaining tables.

A review of the variances indicated the possibility of an error for clover-timothy acres in Crop Reporting District No. 4 (See table 2). The data processing system provided for an output tape with township data on it

including variances within each township. A print out of the township data for CRD No. 4 showed one township that had an extremely large variance, so a print out of individual farm data for this township was called for. The record for one farm showed 5,000 acres of clover-timothy, a record that was clearly in error. The record could have been corrected and the results changed or clover-timothy could have been deleted from the tables. However, results as obtained from the computer output are shown in the tables to illustrate the impact of an error of this kind on the results. Also, it is of interest to consider the impact on sampling error and sample design if in fact one unusual farm did exist that had 5,000 acres of clover-timothy.

Table 1.--State Summary 1/

Item	Total	Farms Reporting		Average Per Farm	Average Per Farm Reporting	Standard Deviation	Relative Variance		Design Efficiency For Ratio Estimator
		Number	Percent				All Farms	Farms Reporting	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Sum		3÷A	2÷A	2÷3	√VAR	7²/5²	4²/6²	VAR OF RATIO / VAR.
Farmland (acres)	17,825,290	101,685	100.0	175.3	175.3	149.3	0.725	0.725	
Population (persons)	423,654	96,428	94.8	4.17	4.39	2.79	0.450	0.372	1.82
Alfalfa (acres)	2,906,718	71,434	70.2	28.6	40.7	32.3	1.277	0.593	.82
All corn (acres)	2,611,523	67,573	66.5	25.7	38.6	46.8	3.33	1.88	.71
All pasture (acres)	3,315,811	62,401	61.4	32.6	53.1	56.9	3.04	1.49	.83
Milk cows (head)	1,657,230	59,728	58.7	16.3	27.7	18.9	1.34	0.37	.98
Beef cattle (head)	595,961	26,895	26.4	5.86	22.2	23.5	16.07	3.50	.93
Clover and timothy (acres)	569,105	19,294	19.0	5.60	29.5	22.2	15.76	2.19	1.00
Hay for silage (acres)	551,600	16,169	15.9	5.42	34.1	19.7	13.20	1.25	.91
Cattle Marketed (head)	188,397	7,600	7.5	1.85	24.8	23.7	163.9	11.36	.98
Soybeans (acres)	126,645	4,125 X	4.1	1.25 O	30.7 X	10.7 O	73.4	1.99 Y	.98
Peas (acres)	101,614	3,180	3.1	1.00	32.0	11.6	134.2	3.24	.97
Stock sheep (head)	77,679	2,742	2.7	.76	28.3	9.0	138.2	2.76	1.00
Spring wheat (acres)	15,281	1,194	1.2	.15	12.8	3.3	488	4.74	.98
Potatoes (acres)	40,079	741	0.7	.39	54.1	11.1	789	4.76	.98
Snap beans (acres)	6,070	234	0.2	.06	25.9	2.3	1501	2.45	1.00

1/ See appendix for explanations keyed to column numbers.

Table 2.--Summary by Crop Reporting Districts 1/

Crop Reporting District (11)	Total (2)	Farms Reporting: Number (3)	Farms Reporting: Percent (4)	Average Per Farm (5)	Average Per Farm Reporting (6)	Standard Deviation (7)	Relative Variance: All Farms (8)	Relative Variance: Farms Reporting (9)	Design Efficiency For Ratio Estimator (10)
				Farmland					
1	1,954,882	10,748	100.0	181.9	181.9	135.0	0.548	0.548	
2	1,873,188	11,166	100.0	167.8	167.8	125.1	0.548	0.548	
3	993,455	5,917	100.0	167.9	167.9	130.2	0.608	0.608	
4	3,070,561	15,342	100.0	200.1	200.1	159.8	0.640	0.640	
5	1,689,508	9,616	100.0	175.7	175.7	190.1	1.166	1.166	
6	2,145,109	15,164	100.0	141.5	141.5	123.5	0.757	0.757	
7	2,826,942	13,654	100.0	207.0	207.0	154.5	0.562	0.562	
8	2,374,689	14,315	100.0	165.9	165.9	148.2	0.792	0.792	
9	896,956	5,763	100.0	155.6	155.6	155.7	1.000	1.000	
STATE	17,825,290	101,685	100.0	175.3	175.3	149.3	0.722	0.722	
				Population					
1	41,815	10,245	95.3	3.89	4.08	2.5	0.410	0.348	1.73
2	46,599	10,683	95.7	4.17	4.36	2.6	0.397	0.336	1.79
3	23,583	5,605	94.7	3.99	4.20	2.6	0.422	0.348	1.82
4	62,807	14,548	94.8	4.09	4.32	2.7	0.422	0.348	1.80
5	36,438	8,982	93.4	3.79	4.06	2.8	0.532	0.422	2.13
6	66,695	14,486	95.5	4.40	4.60	2.8	0.384	0.325	2.23
7	57,849	12,771	93.5	4.24	4.53	2.9	0.462	0.372	1.47
8	63,039	13,669	95.5	4.40	4.61	3.1	0.490	0.422	1.56
9	24,829	5,439	94.4	4.31	4.56	3.2	0.548	0.462	1.75
STATE	423,654	96,428	94.8	4.17	4.39	2.79	0.449	0.372	1.82

1/ See appendix for explanations keyed to column numbers.

Table 2. Con't.--Summary by Crop Reporting Districts 1/

Crop Reporting District	Total	Farms Reporting		Average Per Farm	Average Per Farm Reporting	Standard Deviation	Relative Variance		Design Efficiency For Ratio Estimator
		Number	Percent				All Farms	Farms Reporting	
(11)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
				Alfalfa					
1	236,900	5,502	51.2	22.0	43.1	33.2	2.28	0.67	.76
2	106,157	3,462	31.0	9.51	30.7	19.6	4.24	0.62	.92
3	149,872	3,780	63.9	25.3	39.6	29.5	1.37	0.50	.88
4	531,718	12,146	79.2	34.7	43.8	34.0	0.96	0.55	.72
5	212,840	5,853	60.9	22.1	36.4	28.7	1.69	0.64	1.05
6	543,090	12,893	85.0	35.8	42.1	32.6	0.83	0.56	.82
7	548,712	12,115	88.7	40.2	45.3	33.8	0.71	0.52	.68
8	420,207	11,558	80.7	29.4	36.4	30.3	1.06	0.67	.80
9	157,222	4,125	71.6	27.3	38.1	32.5	1.42	0.72	.79
STATE	2,906,718	71,434	70.2	28.6	40.7	32.3	1.28	0.59	.82
				All Corn					
1	134,005	5,156	48.0	12.5	26.0	23.5	3.53	1.19	.77
2	99,867	5,150	46.1	8.9	19.4	15.3	2.96	0.81	.89
3	86,417	3,289	55.6	14.6	26.3	24.3	2.76	1.08	.90
4	361,254	10,848	70.7	23.5	33.3	31.8	1.82	1.00	.70
5	205,921	5,883	61.2	21.4	35.0	52.2	5.95	3.24	.61
6	374,787	10,556	69.6	24.7	35.5	39.4	2.56	1.46	.66
7	439,529	11,127	81.5	32.2	39.5	41.8	1.69	1.19	.68
8	671,530	11,406	79.7	46.9	58.9	75.4	2.59	1.85	.49
9	238,213	4,158	72.1	41.3	57.3	67.2	2.66	1.64	.49
STATE	2,611,523	67,573	66.5	25.7	38.6	46.8	3.31	1.88	.71

1/ See appendix for explanations keyed to column numbers.

Table 2. Con't.--Summary by Crop Reporting Districts 1/

Crop Reporting District (11)	Total (2)	Farms Reporting: Number (3)	Farms Reporting: Percent (4)	Average Per Farm (5)	Average Per Farm Reporting (6)	Standard Deviation (7)	Relative Variance: All Farms (8)	Relative Variance: Farms Reporting (9)	Design Efficiency For Ratio Estimator (10)
				All Pasture					
1	551,883	7,059	65.7	51.3	78.2	69.8	1.85	0.86	.74
2	444,482	7,796	69.8	39.8	57.0	52.0	1.72	0.88	.88
3	109,614	3,044	51.4	18.5	36.0	34.7	3.50	1.32	.89
4	574,302	10,371	67.6	37.4	55.4	52.2	1.96	1.00	.89
5	220,792	4,578	47.6	23.0	48.2	65.0	7.95	3.28	.87
6	193,157	8,117	53.5	12.7	23.8	20.4	2.56	0.90	1.04
7	806,877	9,976	73.1	59.1	80.9	85.0	2.07	1.23	.67
8	340,653	9,033	63.1	23.8	37.7	39.2	2.72	1.35	.85
9	74,051	2,427	42.1	12.8	30.5	28.0	4.80	1.44	.93
STATE	3,315,811	62,401	61.4	32.6	53.1	56.9	3.06	1.49	.83
				Milk Cows					
1	160,302	6,457	60.1	14.9	24.8	17.4	1.37	0.42	.76
2	194,165	7,766	69.6	17.4	25.0	16.7	0.92	0.34	.91
3	90,887	3,607	61.0	15.4	25.2	17.0	1.21	0.35	.97
4	248,151	9,041	58.9	16.2	27.4	18.5	1.30	0.36	.91
5	110,856	4,775	49.7	11.5	23.2	16.0	1.93	0.46	1.26
6	281,351	9,639	63.6	18.6	29.2	19.3	1.08	0.32	.92
7	242,307	8,511	62.3	17.7	28.5	18.7	1.12	0.31	1.01
8	244,386	7,408	51.7	17.1	33.0	21.7	1.61	0.35	.96
9	84,825	2,524	43.8	14.7	33.6	22.2	2.28	0.44	.91
STATE	1,657,230	59,728	58.7	16.3	27.7	18.9	1.35	0.37	.98

1/ See appendix for explanations keyed to column numbers.

Table 2. Con't.--Summary by Crop Reporting Districts 1/

Crop Reporting District	Total	Farms Reporting		Average Per Farm	Average Per Farm Reporting	Standard Deviation	Relative Variance		Design Efficiency For Ratio Estimator
		Number	Percent				All Farms	Farms Reporting	
(11)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
				Beef Cattle					
1	48,907	3,012	28.0	4.55	16.2	14.3	9.92	2.07	.95
2	26,009	2,189	19.6	2.33	11.9	9.6	16.81	2.50	.98
3	16,433	1,071	18.1	2.78	15.3	12.8	21.25	3.03	.95
4	106,285	4,552	29.7	6.93	23.3	21.4	9.55	2.13	.93
5	39,790	2,193	22.8	4.14	18.1	16.1	15.05	2.66	.93
6	36,839	2,959	19.5	2.43	12.4	11.5	22.37	3.57	.97
7	167,775	4,879	35.7	12.3	34.4	37.8	9.42	2.72	.85
8	128,472	4,767	33.3	8.97	27.0	35.0	15.21	4.41	.91
9	25,451	1,273	22.1	4.42	20.0	19.9	20.25	3.69	.98
STATE	595,961	26,895	26.4	5.86	22.2	23.5	16.08	3.50	.93
				Clover and Timothy					
1	134,353	4,036	37.6	12.5	33.3	24.5	3.84	0.83	.92
2	234,203	7,712	69.1	21.0	30.4	23.3	1.23	0.53	.99
3	45,856	1,422	24.0	7.75	32.2	19.3	6.20	0.72	.95
4	46,317	1,647	10.7	3.02	28.1	41.6	190.16	19.45	1.00
5	56,468	2,228	23.2	5.87	25.3	14.7	6.30	0.69	1.11
6	24,617	918	6.0	1.62	26.8	9.0	30.69	0.92	1.00
7	9,631	396	2.9	.70	24.3	6.2	77.26	1.28	.99
8	7,378	503	3.5	.52	14.7	3.8	55.35	0.98	1.01
9	10,282	432	7.5	1.78	23.8	9.7	29.48	1.30	1.02
STATE	569,105	19,294	19.0	5.60	29.5	22.2	15.76	2.19	1.00

1/ See appendix for explanations keyed to column numbers.

Table 2. Con't.--Summary by Crop Reporting Districts 1/

Crop Reporting District	Total	Farms Reporting		Average Per Farm	Average Per Farm Reporting	Standard Deviation	Relative Variance		Design Efficiency For Ratio Estimator
		Number	Percent				All Farms	Farms Reporting	
(11)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
				Hay for Silage					
1	48,982	1,277	11.9	4.56	38.4	19.6	18.40	1.32	.91
2	52,497	2,025	18.1	4.70	25.9	15.5	10.89	1.14	.91
3	26,414	966	16.3	4.46	27.3	15.4	11.97	1.10	.92
4	81,787	2,466	16.1	5.33	33.2	19.3	13.10	1.28	.90
5	36,245	1,101	11.4	3.77	32.9	16.5	19.18	1.30	.93
6	91,650	2,686	17.7	6.04	34.1	21.1	12.18	1.32	.89
7	90,332	2,552	18.7	6.62	35.4	21.3	10.30	1.12	.91
8	100,219	2,472	17.3	7.00	40.5	23.9	11.70	1.19	.88
9	23,474	624	10.8	4.07	37.6	17.4	18.23	1.08	.92
STATE	551,600	16,169	15.9	5.42	34.1	19.7	13.25	1.25	.91
				Cattle Marketed					
1	3,543	239	2.2	.33	14.8	4.5	187.7	3.20	.99
2	1,660	129	1.2	.15	12.9	3.4	534.1	5.20	1.00
3	1,342	172	2.9	.23	7.8	3.8	281.9	7.24	.99
4	14,603	879	5.7	.95	16.6	8.0	71.0	3.13	.98
5	13,709	650	6.8	1.43	21.1	24.3	288.0	18.49	.94
6	16,999	1,473	9.7	1.12	11.5	12.2	119.5	10.69	.98
7	44,341	938	6.9	3.25	47.3	41.1	159.8	10.05	.98
8	76,664	2,344	16.4	5.36	32.7	39.7	54.8	8.18	.95
9	15,536	776	13.5	2.70	20.0	18.4	46.5	5.43	.98
STATE	188,397	7,600	7.5	1.85	24.8	23.7	164.4	11.36	.98

1/ See appendix for explanations keyed to column numbers.

Table 2. Con't.--Summary by Crop Reporting Districts 1/

Crop Reporting District	Total	Farms Reporting		Average Per Farm	Average Per Farm Reporting	Standard Deviation	Relative Variance		Design Efficiency For Ratio Estimator
		Number	Percent				All Farms	Farms Reporting	
(11)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
				Soybeans					
1	5,029	199	1.9	.47	25.3	7.1	228.6	3.24	.99
2	496	42	0.4	.04	11.8	1.0	557.9	1.10	1.00
3	49	5							
4	31,697	1,221	8.0	2.07	26.0	10.4	25.2	1.08	.96
5	4,773	220	2.3	.50	21.7	4.8	93.7	1.17	.99
6	9,820	353	2.3	.65	27.8	6.6	102.4	1.42	.98
7	4,761	237	1.7	.35	20.1	4.0	127.9	1.25	.99
8	33,327	980	6.8	2.33	34.0	13.5	33.8	1.39	.96
9	36,693	868	15.1	6.37	42.3	30.7	23.1	2.66	.84
STATE	126,645	4,125	4.1	1.25	30.7	10.7	72.8	1.99	.98
				Peas					
1	2,396	74	0.7	.22	32.4	3.6	254.1	0.76	.99
2	1,095	43	0.4	.10	25.5	2.2	517.6	1.00	1.00
3	2,621	66	1.1	.44	39.7	5.5	153.5	0.74	.99
4	5,031	172	1.1	.33	29.2	5.4	274.6	2.07	.99
5	7,517	175	1.8	.78	43.0	9.3	142.8	1.61	.98
6	34,621	1,185	7.8	2.28	29.2	19.1	70.1	4.54	.90
7	3,466	135	1.0	.25	25.7	3.5	189.9	0.88	.99
8	37,785	1,128	7.9	2.64	33.5	19.5	54.3	3.35	.94
9	7,082	202	3.5	1.23	35.1	11.5	87.2	2.10	.98
STATE	101,614	3,180	3.1	1.00	32.0	11.6	134.3	3.24	.97

1/ See appendix for explanations keyed to column numbers.

Table 2. Con't.--Summary by Crop Reporting Districts 1/

Crop Reporting District (11)	Total (2)	Farms Reporting: Number (3)	Farms Reporting: Percent (4)	Average Per Farm (5)	Average Per Farm Reporting (6)	Standard Deviation (7)	Relative Variance: All Farms (8)	Relative Variance: Farms Reporting (9)	Design Efficiency For Ratio Estimator (10)
				Stock Sheep					
1	7,100	220	2.0	.66	32.3	7.9	142.3	1.93	1.00
2	1,916	105	0.9	.17	18.2	3.2	353.1	2.34	1.00
3	1,174	70	1.2	.20	16.8	2.6	174.8	1.08	1.00
4	13,881	405	2.6	.91	34.3	9.1	101.4	1.69	1.00
5	5,003	207	2.2	.52	24.2	5.1	95.4	1.08	1.01
6	4,106	211	1.4	.27	19.5	3.8	198.0	1.77	1.00
7	18,600	627	4.6	1.36	29.7	13.6	100.2	3.65	1.00
8	20,934	717	5.0	1.46	29.2	13.7	88.4	3.46	1.00
9	4,965	180	3.1	.86	27.6	8.2	90.6	1.85	1.00
STATE	77,679	2,742	2.7	.76	28.3	9.0	138.3	2.76	1.00
				Spring Wheat					
1	482	29	0.3	.04	16.6	1.6	1236	2.34	1.00
2	176	16	0.1	.02	11.0	.6	1442	1.06	1.00
3	137	15	0.3	.02	9.1	.7	947	1.42	1.00
4	643	53	0.3	.04	12.1	1.4	1040	2.66	1.00
5	1,610	53	0.6	.17	30.4	8.5	2617	13.40	.98
6	2,663	293	1.9	.18	9.1	1.6	86	0.67	.98
7	338	23	0.2	.02	14.7	.8	1008	0.69	1.00
8	1,546	147	1.0	.11	10.5	1.4	175	0.81	.99
9	7,686	565	9.8	1.33	13.6	6.8	26	1.69	.93
STATE	15,281	1,194	1.2	.15	12.8	3.3	490	4.75	.98

1/ See appendix for explanations keyed to column numbers.

Table 2. Con't.--Summary by Crop Reporting Districts 1/

Crop Reporting District (11)	Total (2)	Farms Reporting		Average Per Farm (5)	Average Per Farm Reporting (6)	Standard Deviation (7)	Relative Variance		Design Efficiency For Ratio Estimator (10)
		Number (3)	Percent (4)				All Farms (8)	Farms Reporting (9)	
				Potatoes					
1	1,737	27	0.3	.16	64.3	5.8	1267	2.19	.99
2	5,524	87	0.8	.50	63.5	11.2	513	3.00	.97
3	7,826	134	2.3	1.32	58.4	19.1	209	3.72	.97
4	698	26	0.2	.05	26.8	1.7	1357	1.30	1.00
5	16,820	245	2.6	1.75	68.7	27.6	248	5.34	.93
6	1,213	43	0.3	.08	28.2	3.8	2227	5.34	1.00
7	642	10	0.1	.05	64.2	3.6	5753	3.20	1.00
8	1,032	31	0.2	.07	33.3	3.6	2431	4.28	1.00
9	4,587	138	2.4	.80	33.2	10.2	166	3.00	.98
STATE	40,079	741	0.7	.39	54.1	11.1	790	4.75	.98
				Snap Beans					
1	1	1							
2	80	1							
3	213	10	.2	.04	21.3	1.1	980	0.66	.99
4	29	8							
5	90	11	0.1	.01	8.2	.4	1940	1.21	1.00
6	1,845	75	0.5	.12	24.6	3.2	669	2.31	1.00
7	23	2							
8	561	32	0.2	.04	17.5	1.7	1981	3.20	1.00
9	3,228	94	1.6	.56	34.3	7.6	185	2.04	.99
STATE	6,070	234	0.2	.06	25.9	2.3	1501	2.46	1.00

1/ See appendix for explanations keyed to column numbers.

Table 3 - Relative Variances for Stratified Random Sampling
Sample Allocation Proportioned to Number of Farms 1/

Item	Mean Estimator (12)					Ratio Estimator (13)			
	Geographic Stratification (14)								
	None	Crop Reporting District	County	Township	Zero Non-Zero	None	Crop Reporting District	County	Township
(1)	(8)	(11)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
Farmland	0.725	0.711	0.698	0.657	0.725	---	---	---	---
Population	0.450	0.447	0.441	0.425	0.392	0.821	0.799	0.783	0.747
Alfalfa	1.277	1.178	1.134	1.024	0.844	1.042	0.948	0.904	0.800
All corn	3.33	3.11	2.98	2.83	2.82	2.35	2.12	2.01	1.89
All pasture	3.04	2.81	2.75	2.15	2.42	2.52	2.37	2.31	1.73
Milk cows	1.34	1.33	1.28	1.19	0.63	1.32	1.28	1.23	1.14
Beef cattle	16.07	15.74	15.52	15.04	13.22	14.92	14.67	14.50	14.06
Clover and timothy	15.76	14.41	14.00	12.93	11.54	15.83	14.45	14.02	12.92
Hay for silage	13.20	13.16	13.07	12.69	7.89	12.00	11.95	11.86	11.52
Cattle marketed	163.9	163.0	162.3	159.6	152.0	160.2	159.3	158.7	156.0
Soybeans	73.4	71.9	69.1	66.5	49.0	71.5	70.0	67.3	64.7
Peas	134.2	133.3	132.0	129.5	103.6	130.8	129.7	128.4	125.9
Stock sheep	138.2	137.8	137.4	136.8	102.2	138.1	137.8	137.4	136.8
Spring wheat	488	484	478	469	405	480	476	470	461
Potatoes	789	787	777	755	652	777	775	765	743
Snap beans	1501	1498	1484	1432	1071	1500	1496	1482	1430

1/ See appendix for explanations keyed to column numbers.

Table 3A - Design Efficiencies for Stratified Random Sampling
Sample Allocation Proportioned to Number of Farms 1/

Item	Mean Estimator (12)					Ratio Estimator (13)			
	Stratification (14)								
	None	Crop Reporting District	County	Township	Zero Non-Zero	None	Crop Reporting District	County	Township
(1)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
Farmland	1.000	.981	.963	.906	1.000	1.000	---	---	---
Population	1.000	.993	.980	.944	.871	1.000	.973	.954	.910
Alfalfa	1.000	.922	.888	.802	.661	1.000	.910	.868	.768
All corn	1.000	.934	.895	.850	.847	1.000	.902	.855	.804
All-pasture	1.000	.924	.905	.707	.796	1.000	.940	.917	.686
Milk cows	1.000	.992	.955	.888	.470	1.000	.970	.932	.864
Beef cattle	1.000	.979	.966	.936	.823	1.000	.983	.972	.942
Clover and timothy	1.000	.914	.888	.820	.732	1.000	.913	.886	.816
Hay for silage	1.000	.997	.990	.961	.598	1.000	.996	.988	.960
Cattle marketed	1.000	.994	.990	.974	.927	1.000	.994	.991	.974
Soybeans	1.000	.980	.941	.906	.668	1.000	.979	.941	.905
Peas	1.000	.993	.984	.965	.772	1.000	.992	.982	.962
Stock sheep	1.000	.997	.994	.990	.740	1.000	.998	.995	.990
Spring wheat	1.000	.992	.980	.961	.830	1.000	.992	.979	.960
Potatoes	1.000	.997	.985	.957	.826	1.000	.997	.984	.956
Snap beans	1.000	.998	.989	.954	.714	1.000	.997	.988	.953
Average for all items except farmland	1.000	.974	.955	.908	.752	1.000	.969	.949	.897

1/ See appendix for explanations keyed to column numbers.

Table 4 - Relative Variances and Design Efficiencies for Alternative Sample Allocations to Crop Reporting Districts 1/

Item	Relative Variance					Design Efficiency		
	Mean Estimator (12)			Ratio Estimator(13)		Mean Estimator (12)		Ratio Estimator(13)
	Allocation Proportional to			Allocation Proportional to				
	Number of Farms	Item Total	Optimum Allocation	Number of Farms	Optimum Allocation	Item Total	Optimum Allocation	Optimum Allocation
(1)	(11)	(31)	(32)	(19)	(33)	(34)	(35)	(36)
Farmland	0.711	---	---	---	---	---	---	---
Population	0.447	0.447	0.445	0.799	0.798	1.000	.996	.999
Alfalfa	1.178	1.228	1.156	0.948	0.925	1.042	.981	.976
All corn	3.11	2.66	2.58	2.12	1.60	.855	.830	.755
All pasture	2.81	2.56	2.42	2.37	1.98	.911	.861	.835
Milk cows	1.33	1.33	1.31	1.28	1.24	1.000	.985	.969
Beef cattle	15.74	13.02	12.71	14.67	11.67	.827	.807	.796
Clover and timothy	14.41	21.90	9.45	14.45	9.38	1.520	.656	.649
Hay for silage	13.16	13.06	12.91	11.95	11.68	.992	.981	.977
Cattle marketed	163.0	111.5	100.3	159.3	97.0	.684	.615	.609
Soybeans	71.9	50.2	41.9	70.0	39.6	.698	.583	.566
Peas	113.3	96.3	87.5	129.7	82.9	.850	.772	.639
Stock sheep	137.8	112.6	109.4	137.8	109.3	.817	.794	.793
Spring wheat	484	451	219	476	214	.932	.452	.450
Potatoes	784	535	405	775	391	.682	.516	.504
Snap beans	1498	740	526	1496	524	.494	.351	.350

1/ See appendix for explanations keyed to column numbers.

Table 5 - Sample Allocation to Crop Reporting Districts for the Mean Estimator 1/

Item	Allocations Proportional to	Sample Allocations to Crop Reporting Districts									Design Efficiency, Mean Estimator
		1	2	3	4	5	6	7	8	9	
(1)	(37)	(38)	(38)	(38)	(38)	(38)	(38)	(38)	(38)	(38)	(39)
	Number of Farms	106	110	58	151	95	149	134	141	57	
Population	Item Total	110	105	56	172	95	120	159	133	50	1.000
	Optimum	95	104	55	145	94	147	139	156	65	.996
Alfalfa	Item Total	81	37	52	183	73	187	189	145	54	1.042
	Optimum	114	70	56	167	88	158	147	139	60	.981
All corn	Item Total	51	38	33	138	79	144	168	257	91	.855
	Optimum	60	41	34	116	120	143	136	257	92	.830
All pasture	Item Total	166	134	33	173	67	58	243	103	22	.911
	Optimum	146	113	40	155	121	60	225	109	31	.861
Milk Cows	Item Total	98	117	55	150	67	170	146	147	51	1.000
	Optimum	99	98	53	149	81	154	134	163	67	.985
Beef Cattle	Item Total	82	44	28	178	67	62	281	216	43	.827
	Optimum	73	50	36	155	73	82	243	236	54	.807
Clover and timothy	Item Total	236	411	81	81	99	43	17	13	18	1.520
	Optimum	151	148	65	365	81	78	48	31	32	.656
Hay for silage	Item Total	89	95	48	148	66	166	164	182	43	.992
	Optimum	106	87	46	149	80	161	147	173	51	.981

1/ See appendix for explanations keyed to column numbers.

Table 5 - Con't. - Sample Allocation to Crop Reporting Districts for the Mean Estimator 1/

Item	Allocations Proportional to	Sample Allocations to Crop Reporting Districts									Design Efficiency, Mean Estimator
		1	2	3	4	5	6	7	8	9	
(1)	(37)	(38)	(38)	(38)	(38)	(38)	(38)	(38)	(38)	(38)	(39)
	Number of Farms	106	110	58	151	95	149	134	141	57	
Cattle Marketed	Item Total	18	9	7	78	73	90	235	407	82	.684
	Optimum	26	20	12	65	124	98	297	301	56	.615
Soybeans	Item Total	40	4	0	250	38	78	38	263	290	.698
	Optimum	93	14	3	195	56	121	66	237	216	.583
Peas	Item Total	24	11	26	50	74	341	34	372	70	.850
	Optimum	40	26	34	88	94	304	50	293	70	.772
Stock sheep	Item Total	91	25	15	179	64	53	239	269	64	.817
	Optimum	104	44	19	172	60	71	229	242	58	.794
Spring wheat	Item Total	31	12	9	42	105	174	22	101	503	.932
	Optimum	75	30	19	92	363	109	48	90	174	.452
Potatoes	Item Total	43	138	195	17	420	30	16	26	114	.682
	Optimum	77	155	140	32	329	71	60	63	73	.516
Snap beans	Item Total	0	13	35	5	15	304	4	92	532	.494
	Optimum	0	61	48	16	28	344	14	175	315	.351

1/ See appendix for explanations keyed to column numbers.

Table 6.--Relative Variances Among Townships 1/

Item	Among Townships Within the State			Among Townships Within CRD's		
	Mean Estimator EPS	Ratio Estimator EPS	Mean Estimator PPS	Mean Estimator EPS	Ratio Estimator EPS	Mean Estimator PPS
(1)	(40)	(41)	(42)	(43)	(44)	(45)
Number of farms	0.508					
Farmland	0.516		0.082			
Population	0.590	0.036	0.030	0.544	0.039	0.029
Alfalfa	0.987	0.356	0.267	0.787	0.226	0.170
All Corn	1.245	0.681	0.529	0.890	0.428	0.339
All pasture	1.757	1.291	0.941	1.537	0.933	0.688
Milk cows	0.841	0.211	0.161	0.772	0.200	0.151
Beef cattle	2.079	1.668	1.208	1.630	1.204	0.953
Clover and timothy	4.184	4.031	3.024	3.183	2.033	1.508
Hay for silage	1.543	0.931	0.659	1.421	0.893	0.662
Cattle marketed	8.66	8.22	6.00	7.53	7.16	5.87
Soybeans	10.05	9.83	7.83	8.60	8.12	6.72
Peas	9.56	8.94	9.49	8.25	7.73	10.18
Stock sheep	4.16	3.86	3.84	3.65	3.38	3.75
Spring wheat	29.1	28.9	25.9	25.8	24.0	20.2
Potatoes	36.1	36.4	51.5	34.8	34.3	43.2
Snap beans	84.3	84.3	98.3	80.6	81.1	91.6

1/ See appendix for explanations keyed to column numbers.

Table 6A.--Design Efficiencies for the Township As a Sampling Unit 1/

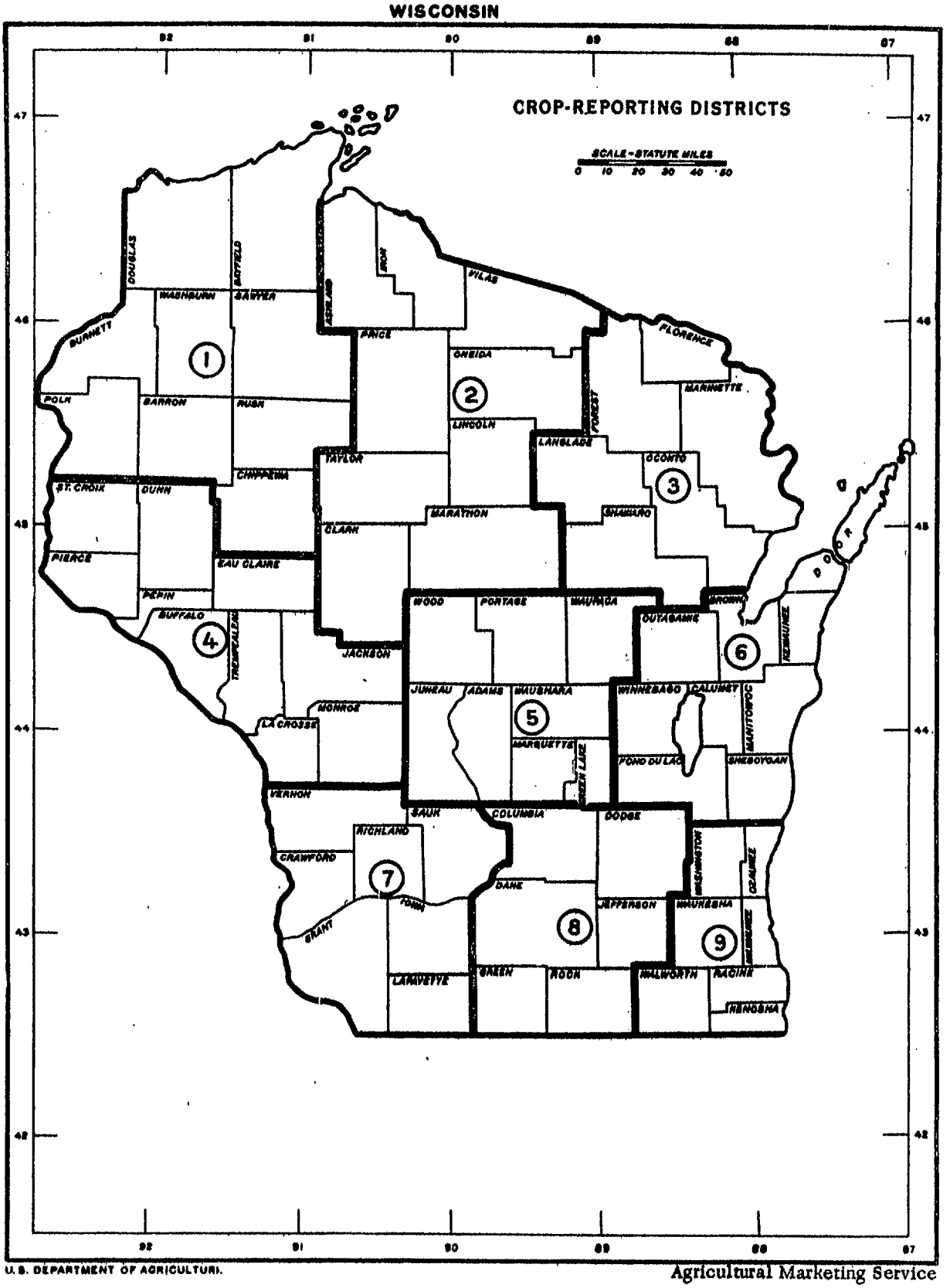
Item	Among Townships Within the State			Among Townships Within CRD's		
	Mean Estimator EPS	Ratio Estimator EPS	Mean Estimator PPS	Mean Estimator EPS	Ratio Estimator EPS	Mean Estimator PPS
(1)	(46)	(47)	(48)	(49)	(50)	(51)
Farmland	1.00		.16	1.00		
Population	1.00	.06	.05	1.00	.07	.05
Alfalfa	1.00	.36	.27	1.00	.29	.22
All corn	1.00	.55	.42	1.00	.48	.38
All pasture	1.00	.74	.54	1.00	.61	.45
Milk cows	1.00	.25	.19	1.00	.26	.20
Beef cattle	1.00	.80	.58	1.00	.74	.58
Clover and timothy	1.00	.96	.72	1.00	.64	.47
Hay for silage	1.00	.60	.43	1.00	.63	.47
Cattle marketed	1.00	.95	.69	1.00	.95	.78
Soybeans	1.00	.98	.78	1.00	.94	.78
Peas	1.00	.94	.99	1.00	.94	1.23
Stock sheep	1.00	.93	.92	1.00	.93	1.03
Spring wheat	1.00	.99	.89	1.00	.93	.78
Potatoes	1.00	1.01	1.43	1.00	.99	1.24
Snap beans	1.00	1.00	1.17	1.00	1.01	1.14
Average for all items except farmland	1.00	.74	.67	1.00	.69	.65

1/ See appendix for explanations keyed to column numbers.

Table 6B.--Design Efficiency of Stratification When the Township Is the Sampling Unit and Design Efficiency of the Township As a Sampling Unit Compared to Individual Farms 1/

Item	Design Efficiency of Stratification by CRD's			Design Efficiency of the Township		
	Mean Estimator EPS	Ratio Estimator EPS	Mean Estimator PPS	No Stratification: Mean Estimator EPS	No Stratification: Mean Estimator PPS	Stratification by CRD's Mean Estimator PPS
(1)	(52)	(53)	(54)	(55)	(56)	(57)
Population	.92	1.08	.97	91.1	4.6	4.5
Alfalfa	.80	.64	.64	53.7	14.5	10.0
All corn	.72	.63	.64	26.0	11.0	7.6
All pasture	.88	.72	.73	40.2	21.5	17.0
Milk cows	.92	.95	.94	43.6	8.3	7.9
Beef cattle	.78	.72	.79	9.0	5.2	4.2
Clover and timothy	.76	.50	.50	18.5	13.3	7.3
Hay for silage	.92	.96	1.01	8.1	3.5	3.5
Cattle marketed	.87	.87	.98	3.7	2.5	2.5
Soybeans	.86	.83	.86	9.5	7.4	6.5
Peas	.86	.87	1.07	5.0	4.9	5.3
Stock sheep	.88	.88	.98	2.1	1.9	1.9
Spring wheat	.89	.83	.78	4.1	3.7	2.9
Potatoes	.96	.94	.84	3.2	4.5	3.8
Snap beans	.96	.96	.93	3.9	4.6	4.2
Average for all items	.86	.83	.84			

1/ See appendix for explanations keyed to column numbers.



U.S. DEPARTMENT OF AGRICULTURE

Agricultural Marketing Service

Figure 1

APPENDIX A

Explanations of Tables

Numbers in parenthesis are keyed to columns in the tables. In the formula, upper case letters refer to the population and lower case letters to a sample.

(1) The items are listed in order of decreasing percent reporting, column (4), primarily because sampling variance and variability patterns are closely related to percent reporting.

(2) This column shows State totals, table 1, and totals by CRD's, table 2. The data are totals as enumerated in the Wisconsin State Farm Census for 1970 and are not official estimates.

(3) The term "farms reporting" denotes the number of farms having the item. If X_i is the value of item X for the ith farm, then the number of "farms reporting" this item is the number of farms having a value of X_i greater than zero. All farms have some farmland so the number of farms reporting farmland is the total number of farms. The number of farms reporting population is less than the number of farms because there are no farm families living on some farms.

(4) This column shows the number of farms reporting as a percent of all farms. It is column (3) divided by the total number of farms 101,685 and expressed in percent.

(5) This is column (2) divided by the total number of farms.

(6) Column (6) is column (2) divided by column (3).

(7) The standard deviation is $S_X = \sqrt{\frac{\Sigma (X_i - \bar{X})^2}{N-1}}$ where X_i is the value of the variable X for the ith farm and N is the total number of farms.

In table 1 the standard deviation measures the variation among all farms in the State. In table 2 the standard deviation among farms within each crop reporting district is shown. The last line for each commodity in table 2 is for the whole State and is the same as the data shown in table 1.

(8) Relative variance is $\frac{S_x^2}{\bar{X}^2}$. It is the variance, square of the standard deviation, divided by the square of the average per farm. One may derive column (8) by taking the square of each entry in column 7 and dividing by the square of the corresponding entry in column (5).

(9) Relative variance for farms reporting is analogous to the relative variance for all farms. It is the relative variance among farms in a subset of farms reporting. For example, for soybeans it is the variance among the 4,125 farms reporting soybeans divided by 30.7^2 which is the square of the average number of acres of soybeans on the 4,125 farms.

(10) Design efficiency denotes the ratio of two variances for the purpose of showing the size of the variance for one method in comparison to another. In table 1, column (10) is the ratio of the variance for the ratio estimator $\left(\sum Y_i\right)\left(\frac{\bar{x}}{\bar{y}}\right)$ to the variance for the mean estimator $N\bar{x}$ assuming simple random sampling, where $\bar{x}$ is the sample average for the item, $\bar{y}$ is the sample average for farmland, N is the total number of farms, and $\sum Y_i$ is total farmland. In table 2 the comparison is made for each Crop Reporting District.

(11) See figure 1 for a definition of Crop Reporting Districts. As can be seen to some extent from table 2, there is a substantial difference in agriculture between the northern and southern parts of the State.

(12) For simple random sampling the mean estimator is $N\bar{x}$. For stratified random sampling the mean estimator is $\sum N_h \bar{x}_h$ where N_h is the total number of farms in stratum h and $\bar{x}_h$ is the sample average per farm for stratum h.

(13) For simple random sampling the ratio estimator is $\left(\sum Y_i\right)\left(\frac{\bar{x}}{\bar{y}}\right)$. For stratified random sampling the ratio estimator is $\sum Y_i \frac{\sum N_h \bar{x}_h}{\sum N_h \bar{y}_h}$.

(14) There were four levels of geographic stratification: None, Crop Reporting Districts, Counties, and Townships. There were 9 CRD's, 72 counties, and 1,462 townships. Table 3 presents average within stratum variances which are applicable for stratified random sampling of individual farms when the sample is selfweighting, that is, allocated to strata in proportion to the total number of farms. The relative variances recorded in table 3 are $V^2 = \frac{N(\sum N_h S_h^2)}{(\sum X_i)^2} = \frac{(\sum N_h S_h^2)}{(\bar{X})^2}$ where h denotes a stratum,

N_h is the number of farms in stratum h,

$N = \sum N_h$ the total number of farms in the State,

S_h^2 is the variance within stratum h, and

$\sum X_i$ = State total of the item.

For the mean estimator, $S_h^2 = \frac{\sum_{i=1}^{N_h} (X_{hi} - \bar{X}_h)^2}{N_h - 1}$

For the ratio estimator, $S_h^2 = \frac{\sum_{i=1}^{N_h} (X_{hi} - RY_{hi})^2}{N_h - 1}$ where $R = \frac{\sum X_i}{\sum Y_i}$.

v^2 is the relative variance among individual farms within strata and may be thought of as the variance for a sample of one farm even though it is impossible to select a stratified random sample of one farm. The variance for another ratio estimator $\sum Y_h \frac{\bar{x}_h}{\bar{y}_h}$ was also computed, where Y_h is the total farmland in stratum h. Its variance was either identical to or differed by a trivial amount from the variance of $\left(\sum^{N} Y_i\right)\frac{\sum N_h \bar{x}_h}{\sum N_h \bar{y}_h}$.

(15) See (14).

(16) See (14).

(17) This column in table 3 is of interest in comparison with columns (8) and (9). It shows what the relative variance for a mean estimator would be if one had proportionate representation between two strata: farms reporting, and farms having none of the item. These two strata differ from item to item and no geographic stratification is assumed.

Mathematically, the mean estimator and its relative variance are as follows:

The estimator is $N_o \bar{x}_o + N_r \bar{x}_r = N_r \bar{x}_r$

where N_r = number of farms reporting the item

$\bar{x}_r$ = average per farm reporting

N_o = number of farms not having the item

$\bar{x}_o$ = zero

The relative variance column (17) is $\frac{N N_r S_r^2}{(\sum X_i)^2}$ where r refers to the stratum of farms reporting.

Excepting rounding error, column (17) of table 3 can be derived by dividing the entries in column (9) of table 1 by the corresponding entries in column (4) of table 1.

(18) See (14).

(19) See (14).

(20) See (14).

(21) See (14).

(22) thru (30). These columns, table 3A, correspond to columns in table 3. Table 3A shows design efficiencies for the mean estimator which are obtained by dividing columns (8), (11), (15), (16), and (17) by column (8); and design efficiencies for the ratio estimator which are obtained by dividing columns (18), (19), (20), and (21) by column (18). If the ratio estimator had been more efficient relative to the mean estimator, the reduction in variance due to stratification probably would have been quite different for the two estimators.

(31) This column shows the relative variances for stratified random sampling and the mean estimator when the sample is allocated to Crop Reporting Districts in proportion to the item total. The relative variances recorded are $\frac{1}{(\Sigma X)^2}\left[\Sigma \frac{N_h^2 S_h^2}{P_h}\right]$ where P_h is the proportion of the item in stratum h. The allocation differs for every item. Table 5 shows allocations assuming a sample of 1,000 farms, even though the variances in table 4 are expressed on a unit basis and can be interpreted as variances for a hypothetical sample of one farm.

(32) For the mean estimator and stratification by CRD's the sampling variance is a minimum when the sample is allocated so $n_h \propto N_h S_h$ where n_h is the sample size for stratum h. The relative variance for this allocation is

recorded in column (32). It is $\frac{1}{(\Sigma X)^2}\left[\Sigma \frac{N_h^2 S_h^2}{P_h'}\right]$ where $P_h' = \frac{N_h S_h}{\Sigma N_h S_h}$.

In this case $\frac{1}{(\Sigma X)^2}\left[\Sigma \frac{N_h^2 S_h^2}{P_h'}\right]$ reduces to $\frac{(\Sigma N_h S_h)^2}{(\Sigma X)^2}$.

(33) The optimum allocation for the ratio estimator is analogous to the optimum for the mean estimator. Simply substitute in (32) the values of S_h^2 for the ratio estimator. Optimum allocations for the ratio estimator are not shown in table 5 because they are very close to the optimum allocations for the mean estimator.

(34) Column (34) is column (31) divided by column (11).

(35) Column (35) is column (32) divided by column (11).

(36) Column (36) is column (33) divided by column (19).

(37) This column shows the basis for allocation. See (31) and (32).

(38) For the various items and criteria for allocation, the columns identified as (38) show sample sizes by CRD's (i.e. strata) for a total sample size of 1,000.

(39) The numbers in this column come from columns (34) and (35) of table 4. This column facilitates looking at differences in the sample allocations and observing the impact on sampling variance.

(40) EPS denotes "equal probability of selection." Hence, the relative variances in this column are for an unstratified random sample of townships using equal probability of selection and a mean estimator. The mean estimator in this case is the average per township multiplied by the total number of townships. Let T represent a township total for one of the items. The relative variance in column (40) is $V_T^2 = \frac{M^2}{(\Sigma T)^2}\left[\frac{\Sigma (T-\bar{T})^2}{M-1}\right]$ where M is the total number of townships and ΣT is the State total for the item.

Note from the formula for V_T^2 that the relative variances in table 6 are expressed in terms of one township, that is, the township is the sampling unit and is enumerated completely. The average number of farms in a township is 69.5. The variances in table 6 must be multiplied by 69.5 to express them on a per farm basis and to make them comparable with the variances in tables 1, 2, 3, and 4.

(41) The specifications are the same as for (40) except that a ratio estimator is used. There are two possible ratio estimators: $N\frac{\Sigma x}{n}$ or $\Sigma Y\frac{\Sigma x}{\Sigma y}$ where N is the total number of farms in the State,

Σx is the sample total,

n is the total number of farms in the sample. (Note that n is a random variable in this case.)

ΣY is the total farmland in the State, and

Σy is the total farmland in the sample.

The variances for these two estimators were very close to being the same so only the relative variances for $N\frac{\Sigma x}{n}$ are presented, column (41).

(42) PPS denotes "probability proportional to size." Size in this case was number of farms. For a sample of m townships selected with replacement, the mean estimator involves a weighted mean. It is $\frac{M}{m}\Sigma\frac{t}{P}$ where t is the township total for a township in the sample and P is its probability of selection. The relative variances recorded in this column are $\frac{\Sigma P_i\left[\frac{T_i}{P_i}-\Sigma T_i\right]^2}{(\Sigma T_i)^2}$ where i refers to township, T is the township total for an item, and $P_i = \frac{N_i}{N}$ where N_i equals the number of farms in the ith township.

(43), (44), and (45) These columns correspond, respectively, to columns (40), (41), and (42). The only difference is that stratification by CRD's is imposed.

(46) thru (51) These columns correspond to columns (40) thru (45) of table 6. They show design efficiencies for alternative methods of estimation and probabilities of selection. Columns (46), (47), and (48) are equal to columns (40), (41), and (42) divided by column (40). Columns (49), (50), and (51) are equal to columns (43), (44), and (45) divided by column (43).

(52), (53), and (54) These columns show the design efficiency attributable to stratification by crop reporting districts. Column (52) is column (43) divided by column (40); (53) is (44) divided by (41); and (54) is (45) divided by (42). One of many comparisons of interest is the comparison of columns (52), (53), and (54) with columns (23) and (28) in table 3A.

(55) This column illustrates the loss of efficiency (or increase in sampling variance) owing to variation in the size of township and intraclass correlation when the sampling units are townships instead of individual farms. The average township had 69.5 farms. Column (55) is 69.5 times $\frac{\text{Col. (40)}}{\text{Col. (8)}}$. To illustrate, the 26.0 for corn means, if a township is used as a sampling, that the number of farms must be 26 times larger to have the same sampling error.

(56) This column is 69.5 $\frac{\text{Col. (42)}}{\text{Col. (18)}}$. Selection of townships with probability proportional to number of farms has the effect of reducing the variation among townships which is associated with variation in number of farms.

(57) This column is 69.5 $\frac{\text{Col. (45)}}{\text{Col. (11)}}$. It is analogous to column (56). The entries in column (57) tend to be less than the entries in column (56) because stratification has a greater impact on the variation among townships than on the variation among farms.